To simplify an nth root, identify factors in the radicand with exponents that are multiples of n.

MULTIPLYING AND SIMPLIFYING

We have used the product rule for radicals to find products and also to simplify radical expressions. For some radical expressions, it is possible to do both: First find a product and then simplify.

EXAMPLE 5 Multiply and simplify.

a) $\sqrt{15}\sqrt{6}$ **b)** $3\sqrt[3]{25} \cdot 2\sqrt[3]{5}$ **c)** $\sqrt[4]{8x^3y^5}\sqrt[4]{4x^2y^3}$

SOLUTION

a) $\sqrt{15}\sqrt{6} = \sqrt{15 \cdot 6}$ Multiplying radicands

$= \sqrt{90} = \sqrt{9 \cdot 10}$ 9 is a perfect square.

$= 3\sqrt{10}$

b) $3\sqrt[3]{25} \cdot 2\sqrt[3]{5} = 3 \cdot 2 \cdot \sqrt[3]{25 \cdot 5}$ Using a commutative law; multiplying radicands

$= 6 \cdot \sqrt[3]{125}$ 125 is a perfect cube.

$= 6 \cdot 5$, or 30

c) $\sqrt[4]{8x^3y^5}\sqrt[4]{4x^2y^3} = \sqrt[4]{32x^5y^8}$ Multiplying radicands

$= \sqrt[4]{16x^4y^8 \cdot 2x}$ Identifying the largest perfect fourth-power factor

$= \sqrt[4]{16x^4y^8}\sqrt[4]{2x}$ Factoring into radicals

$= 2xy^2\sqrt[4]{2x}$ Finding the fourth roots; assume $x \geq 0$.

The checks are left to the student.

Try Exercise 65.

Teaching Tip

Consider doing an example like $\sqrt{2x^8}\sqrt{9x^3}$ in two ways. First, simplify and multiply. Next, multiply first and then simplify. This can be the basis for discussing the best way to do these problems.

Student Notes

To multiply $\sqrt{x} \cdot \sqrt{x}$, remember what $\sqrt{x}$ represents and go directly to the product, x. For $x \geq 0$,

$\sqrt{x} \cdot \sqrt{x} = x$,

$(\sqrt{x})^2 = x$, and

$\sqrt{x^2} = x$.

7.3 Exercise Set

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Concept Reinforcement *Classify each of the following statements as either true or false.*

1. For any real numbers $\sqrt[n]{a}$ and $\sqrt[n]{b}$, $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$. True
2. For any real numbers $\sqrt[n]{a}$ and $\sqrt[n]{b}$, $\sqrt[n]{a} + \sqrt[n]{b} = \sqrt[n]{a + b}$. False
3. For any real numbers $\sqrt[n]{a}$ and $\sqrt[m]{b}$, $\sqrt[n]{a} \cdot \sqrt[m]{b} = \sqrt[nm]{ab}$. False
4. For $x > 0$, $\sqrt{x^2 - 9} = x - 3$. False
5. The expression $\sqrt[3]{X}$ is not simplified if X contains a factor that is a perfect cube. True
6. It is often possible to simplify $\sqrt{A \cdot B}$ even though $\sqrt{A}$ and $\sqrt{B}$ cannot be simplified. True

Multiply.

7. $\sqrt{5}\sqrt{7}$ $\sqrt{35}$
8. $\sqrt{10}\sqrt{3}$ $\sqrt{30}$
9. $\sqrt[3]{3}\sqrt[3]{2}$ $\sqrt[3]{6}$
10. $\sqrt[3]{2}\sqrt[3]{5}$ $\sqrt[3]{10}$
11. $\sqrt[4]{6}\sqrt[4]{3}$ $\sqrt[4]{18}$
12. $\sqrt[4]{8}\sqrt[4]{9}$ $\sqrt[4]{72}$

13. $\sqrt{2x}\sqrt{13y}$ $\sqrt{26xy}$
14. $\sqrt{5a}\sqrt{6b}$ $\sqrt{30ab}$ Aha!
15. $\sqrt[5]{8y^3}\sqrt[5]{10y}$ $\sqrt[5]{80y^4}$
16. $\sqrt[5]{9t^2}\sqrt[5]{2t}$ $\sqrt[5]{18t^3}$
17. $\sqrt{y-b}\sqrt{y+b}$ $\sqrt{y^2-b^2}$
18. $\sqrt{x-a}\sqrt{x+a}$ ⊡
19. $\sqrt[3]{0.7y}\sqrt[3]{0.3y}$ $\sqrt[3]{0.21y^2}$
20. $\sqrt[3]{0.5x}\sqrt[3]{0.2x}$ ⊡
21. $\sqrt[5]{x-2}\sqrt[5]{(x-2)^2}$ $\sqrt[5]{(x-2)^3}$
22. $\sqrt[4]{x-1}\sqrt[4]{x^2+x+1}$ $\sqrt[4]{x^3-1}$
23. $\sqrt{\frac{3}{t}}\sqrt{\frac{7s}{11}}$ $\sqrt{\frac{21s}{11t}}$
24. $\sqrt{\frac{5x}{6}}\sqrt{\frac{11}{y}}$ $\sqrt{\frac{55x}{6y}}$
25. $\sqrt[7]{\frac{x-3}{4}}\sqrt[7]{\frac{5}{x+2}}$ $\sqrt[7]{\frac{5x-15}{4x+8}}$
26. $\sqrt[6]{\frac{a}{b-2}}\sqrt[6]{\frac{3}{b+2}}$ $\sqrt[6]{\frac{3a}{b^2-4}}$

Simplify by factoring.

27. $\sqrt{18}$ $3\sqrt{2}$
28. $\sqrt{50}$ $5\sqrt{2}$
29. $\sqrt{27}$ $3\sqrt{3}$
30. $\sqrt{45}$ $3\sqrt{5}$
31. $\sqrt{8x^9}$ $2x^4\sqrt{2x}$
32. $\sqrt{75y^5}$ $5y^2\sqrt{3y}$
33. $\sqrt{120}$ $2\sqrt{30}$
34. $\sqrt{350}$ $5\sqrt{14}$
35. $\sqrt{36a^4b}$ $6a^2\sqrt{b}$
36. $\sqrt{175y^8}$ $5y^4\sqrt{7}$
37. $\sqrt[3]{8x^3y^2}$ $2x\sqrt[3]{y^2}$
38. $\sqrt[3]{27ab^6}$ $3b^2\sqrt[3]{a}$
39. $\sqrt[3]{-16x^6}$ $-2x^2\sqrt[3]{2}$
40. $\sqrt[3]{-32a^6}$ $-2a^2\sqrt[3]{4}$

Find a simplified form of $f(x)$. Assume that x can be any real number.

41. $f(x) = \sqrt[3]{125x^5}$ ⊡
42. $f(x) = \sqrt[3]{16x^6}$ ⊡
43. $f(x) = \sqrt{49(x-3)^2}$ ⊡
44. $f(x) = \sqrt{81(x-1)^2}$ ⊡
45. $f(x) = \sqrt{5x^2-10x+5}$ $f(x) = |x-1|\sqrt{5}$
46. $f(x) = \sqrt{2x^2+8x+8}$ $f(x) = |x+2|\sqrt{2}$

Simplify. Assume that no radicands were formed by raising negative numbers to even powers.

47. $\sqrt{a^6b^7}$ $a^3b^3\sqrt{b}$
48. $\sqrt{x^6y^9}$ $x^3y^4\sqrt{y}$
49. $\sqrt[3]{x^5y^6z^{10}}$ $xy^2z^3\sqrt[3]{x^2z}$
50. $\sqrt[3]{a^6b^7c^{13}}$ $a^2b^2c^4\sqrt[3]{bc}$
51. $\sqrt[4]{16x^5y^{11}}$ $2xy^2\sqrt[4]{xy^3}$
52. $\sqrt[5]{-32a^7b^{11}}$ $-2ab^2\sqrt[5]{a^2b}$
53. $\sqrt[5]{x^{13}y^8z^{17}}$ $x^2yz^3\sqrt[5]{x^3y^3z^2}$
54. $\sqrt[5]{a^6b^8c^9}$ $abc\sqrt[5]{ab^3c^4}$ TW
55. $\sqrt[3]{-80a^{14}}$ $-2a^4\sqrt[3]{10a^2}$
56. $\sqrt[4]{810x^9}$ $3x^2\sqrt[4]{10x}$

Multiply and simplify. Assume that no radicands were formed by raising negative numbers to even powers.

57. $\sqrt{6}\sqrt{3}$ $3\sqrt{2}$
58. $\sqrt{15}\sqrt{5}$ $5\sqrt{3}$
59. $\sqrt{10}\sqrt{14}$ $2\sqrt{35}$
60. $\sqrt{6}\sqrt{33}$ $3\sqrt{22}$
61. $\sqrt[3]{9}\sqrt[3]{3}$ 3
62. $\sqrt[3]{2}\sqrt[3]{4}$ 2

⊡ Answers to Exercises 18, 20, 41–44, and 68 are on p. IA-15.

63. $\sqrt{18a^3}\sqrt{18a^3}$ $18a^3$
64. $\sqrt{75x^7}\sqrt{75x^7}$ $75x^7$
65. $\sqrt[3]{5a^2}\sqrt[3]{2a}$ $a\sqrt[3]{10}$
66. $\sqrt[3]{7x}\sqrt[3]{3x^2}$ $x\sqrt[3]{21}$
67. $3\sqrt{2x^5}\cdot 4\sqrt{10x^2}$ $24x^3\sqrt{5x}$
68. $3\sqrt{5a^7}\,2\sqrt{15a^3}$ ⊡
69. $\sqrt[3]{s^2t^4}\sqrt[3]{s^4t^6}$ $s^2t^3\sqrt[3]{t}$
70. $\sqrt[3]{x^2y^4}\sqrt[3]{x^2y^6}$ $xy^3\sqrt[3]{xy}$
71. $\sqrt[3]{(x+5)^2}\sqrt[3]{(x+5)^4}$ $(x+5)^2$
72. $\sqrt[3]{(a-b)^5}\sqrt[3]{(a-b)^7}$ $(a-b)^4$
73. $\sqrt[4]{20a^3b^7}\sqrt[4]{4a^2b^5}$ $2ab^3\sqrt[4]{5a}$
74. $\sqrt[4]{9x^7y^2}\sqrt[4]{9x^2y^9}$ $3x^2y^2\sqrt[4]{xy^3}$
75. $\sqrt[5]{x^3(y+z)^6}\sqrt[5]{x^3(y+z)^4}$ $x(y+z)^2\sqrt[5]{x}$
76. $\sqrt[5]{a^3(b-c)^4}\sqrt[5]{a^7(b-c)^4}$ $a^2(b-c)\sqrt[5]{(b-c)^3}$
77. TW Explain how you could convince a friend that $\sqrt{x^2-16} \neq \sqrt{x^2}-\sqrt{16}$.
78. TW Why is it incorrect to say that, in general, $\sqrt{x^2} = x$?

SKILL REVIEW

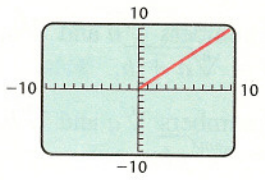
Review simplifying rational expressions (Sections 6.1–6.3).

Perform the indicated operation and, if possible, simplify.

79. $\frac{15a^2x}{8b}\cdot\frac{24b^2x}{5a}$ [6.1] $9abx^2$
80. $\frac{x^2-1}{x^2-4}\div\frac{x^2-x-2}{x^2+x-2}$ [6.1] $\frac{(x-1)^2}{(x-2)^2}$
81. $\frac{x-3}{2x-10}-\frac{3x-5}{x^2-25}$ [6.2] $\frac{x+1}{2(x+5)}$
82. $\frac{6x}{25y^2}+\frac{3y}{10x}$ [6.2] $\frac{3(4x^2+5y^3)}{50xy^2}$
83. $\frac{a^{-1}+b^{-1}}{ab}$ [6.3] $\frac{b+a}{a^2b^2}$
84. $\frac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x}+\frac{1}{x+1}}$ [6.3] $\frac{-x-2}{4x+3}$

SYNTHESIS

85. Abdul is puzzled. When he uses a graphing calculator to graph $y = \sqrt{x}\cdot\sqrt{x}$, he gets the following screen. Explain why Abdul did not get the complete line $y = x$.



86. Is the equation $\sqrt{(2x+3)^8} = (2x+3)^4$ always, sometimes, or never true? Why?

87. *Radar Range.* The function given by

$$R(x) = \frac{1}{2}\sqrt[4]{\frac{x \cdot 3.0 \times 10^6}{\pi^2}}$$

can be used to determine the maximum range $R(x)$, in miles, of an ARSR-3 surveillance radar with a peak power of x watts. Determine the maximum radar range when the peak power is 5×10^4 watts.
Source: Introduction to RADAR Techniques, Federal Aviation Administration, 1988 175.6 mi

88. *Speed of a Skidding Car.* Under certain conditions, police can estimate the speed at which a car was traveling by measuring its skid marks. The function given by

$$r(L) = 2\sqrt{5L}$$

can be used, where L is the length of a skid mark, in feet, and $r(L)$ is the speed, in miles per hour. Find the exact speed and an estimate (to the nearest tenth mile per hour) for the speed of a car that left skid marks **(a)** 20 ft long; **(b)** 70 ft long; **(c)** 90 ft long. See also Exercise 101.

89. *Wind Chill Temperature.* When the temperature is T degrees Celsius and the wind speed is v meters per second, the *wind chill temperature*, T_w, is the temperature (with no wind) that it feels like. Here is a formula for finding wind chill temperature:

$$T_w = 33 - \frac{(10.45 + 10\sqrt{v} - v)(33 - T)}{22}.$$

Estimate the wind chill temperature (to the nearest tenth of a degree) for the given actual temperatures and wind speeds.

a) $T = 7°C$, $v = 8$ m/sec $-3.3°C$
b) $T = 0°C$, $v = 12$ m/sec $-16.6°C$
c) $T = -5°C$, $v = 14$ m/sec $-25.5°C$
d) $T = -23°C$, $v = 15$ m/sec $-54.0°C$

Simplify. Assume that all variables are nonnegative.

90. $\left(\sqrt{r^3t}\right)^7$ $r^{10}t^3\sqrt{rt}$

91. $\left(\sqrt[3]{25x^4}\right)^4$ $25x^5\sqrt[3]{25x}$

92. $\left(\sqrt[3]{a^2b^4}\right)^5$ $a^3b^6\sqrt[3]{ab^2}$

93. $\left(\sqrt{a^3b^5}\right)^7$ $a^{10}b^{17}\sqrt{ab}$

Draw and compare the graphs of each group of functions.

94. $f(x) = \sqrt{x^2 + 2x + 1}$,
$g(x) = x + 1$,
$h(x) = |x + 1|$

95. $f(x) = \sqrt{x^2 - 2x + 1}$,
$g(x) = x - 1$,
$h(x) = |x - 1|$

96. If $f(t) = \sqrt{t^2 - 3t - 4}$, what is the domain of f?

97. What is the domain of g, if $g(x) = \sqrt{x^2 - 6x + 8}$? $\{x \mid x \le 2 \text{ or } x \ge 4\}$, or $(-\infty, 2] \cup [4, \infty)$

Solve.

98. $\sqrt[3]{5x^{k+1}}\sqrt[3]{25x^k} = 5x^7$, for k 10

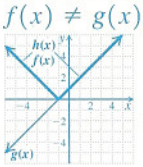
99. $\sqrt[5]{4a^{3k+2}}\sqrt[5]{8a^{6-k}} = 2a^4$, for k 6

100. Use a graphing calculator to check your answers to Exercises 21 and 41.

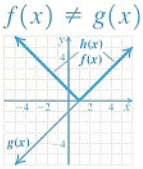
101. Does a car traveling twice as fast as another car leave a skid mark that is twice as long? (See Exercise 88.) Why or why not?

Try Exercise Answers: Section 7.3
7. $\sqrt{35}$ **31.** $2x^4\sqrt{2x}$ **45.** $f(x) = |x - 1|\sqrt{5}$
51. $2xy^2\sqrt[4]{xy^3}$ **65.** $a\sqrt[3]{10}$

88. **(a)** 20 mph; **(b)** $10\sqrt{14} \approx 37.4$ mph; **(c)** $30\sqrt{2} \approx 42.4$ mph
94. $f(x) = h(x)$; $f(x) \neq g(x)$



95. $f(x) = h(x)$; $f(x) \neq g(x)$



96. $\{t \mid t \le -1 \text{ or } t \ge 4\}$, or $(-\infty, -1] \cup [4, \infty)$